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Thermomechanical Deformation in an Orthotropic Micropolar Thermoelastic Solid

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Abstract The thermomechanical deformation in an orthotropic micropolar generalized thermoelastic half-space is investigated. Descarte's method, along with the irreducible case of Cardon's method, is used to obtain the roots of an eight-degree equation. Laplace and Fourier transform techniques are used to obtain the general solution for the set of boundary value problems. Particular types of boundary conditions have been taken to illustrate the utility of the approach. The transformed components of the stresses and temperature distribution have been obtained. A numerical inversion technique is employed to invert the integral transform, and the resulting quantities are presented graphically.

Keywords Orthotropic micropolar thermoelastic solid · Relaxation time · Half-space

1 Introduction

The classical continuum theory was based on the assumption that all material bodies possess continuous mass densities, independent of how small they might be. However, the molecular theory of matter had shown that when the volume became smaller than a certain limit, the material body behaved quite differently. As a result, the classical continuum theory might no longer serve as an appropriate mathematical model when the length scale was comparable to the average grain or molecular size contained in the body. If the physical phenomenon under study had a certain characteristic length,

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which was comparable to the size of the grains in the body, the microstructure of the material must be considered.

Micropolar elasticity theory, which takes into consideration the granular character of the medium, describes deformation by a microrotation and a macrodisplacement. Eringen first showed that the classical elasticity theory [1] and the coupled stress theory [2] are two special cases of the micropolar elasticity theory. The coupled theory of classical thermoelasticity consists of two equations. One of them is the partial differential equation of motion, and the other is the heat conduction equation. The latter equation is of a diffusion type, predicting infinite speeds of propagation for thermal signals, which is physically absurd. To overcome this drawback, generalized theories of thermoelasticity were developed.

The first was developed by Lord and Shulman [3], who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier law. This new law contains the heat flux vector and its time derivative. It also contains a new constant that acts as a relaxation time. The remaining governing equations for this theory, namely, the equation of motion and the constitutive relations, remain the same as those for coupled and uncoupled theories. The second was developed by Green and Lindsay [4]. This theory contains two relaxation times and modifies not only the heat conduction equation, but also all the equations of coupled theory. The two theories ensure a finite speed of propagation of a heat wave. These theories were extended by Dhaliwal and Sherief [5] to general anisotropic media in the presence of heat sources.

The dynamic response function of elastically anisotropic solids is of interest in many fields including crystal acoustics, solid-state physics, non-destructive testing, material characterization, seismology, applied mechanics, and mathematics. In recent years, the elastodynamic response of an anisotropic continuum has received the attention of several studies. Wang and Achenbach [6] obtained a two-dimensional time domain elastodynamic displacement Green's function for general anisotropic solids. Wu [7] provided an explicit solution for the surface displacements due to an impulsive line source within a general anisotropic half-space.

The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects by Nowacki [8] and Eringen [9]. Tauchert et al. [10] also derived the basic equations of the linear theory of micropolar thermoelasticity. Dost and Tabarrok [11] presented the theory of micropolar generalized thermoelasticity by using the Green-Lindsay theory. Chandrasekhariah [12] formulated a theory of micropolar thermoelasticity which includes a heat-flux vector among the constitutive variables. Martynenko and Bosyakov [13] investigated a surface of discontinuity for a cubically anisotropic body in the theory of micropolar thermoelasticity. Hasan and Dyszlewicz [14,15] discussed some problems in the theory of micropolar thermoelasticity. Tian-Min [16] restudied the coupled field theories for micropolar continua and micropolar thermoelasticity. Martynenko and Bosyakov [17] investigated the wave process in a thermoelastic micropolar solid body by the method of the theory of characteristics. Recently, Kumar and Ailawalia [18–21] discussed various problems in micropolar thermoelasticity.

Iesan [22] studied the static theory of anisotropic micropolar elastic solids and proved the positive definiteness of his operator for the first boundary value problem. Kumar and Choudhary [23–25] discussed various problems in an orthotropic micropolar continuum. However, no attempt has been made to study the thermal effect in an orthotropic micropolar solid due to various sources.

The potential function approach is often used by many investigators to solve various problems. This however, has several disadvantages as outlined in [26]. These may be summarized in the fact that the boundary conditions that naturally arise in a typical boundary value problem are directly related to the actual physical quantities that appear in the governing equations, and not the potential function associated with the problem. Thus, the introduction of an auxiliary potential function in formulating the problems appears artificial and not physically motivated and, hence, not intuitively appealing, despite the simplification it brings about in the theoretical development and discussion. Secondly, a more stringent assumption must be made on the behavior of the potential function than on the actual physical quantities. Last of all, it was found that many integral representations of physical quantities are convergent in the classical sense, while the potential functions from which they are derivable only converge in a distributional sense. Because of all these reasons, many researchers avoided the use of potential functions. Among the alternatives is the approach that we have used in the present paper.

In high-temperature applications, thermal stresses generated from a heat temperature increase and cooling processes may rise above the ultimate strength and lead to unexpected failures. Thus, the importance of the thermal stresses in causing structural damage and changes in functionality of the structure is well recognized whenever thermal environments are involved. Therefore, the ability to predict elastodynamic stresses induced by sudden thermal loading in a composite structure is essential for proper and safe design and the knowledge of its response during service in these thermal applications. For the case of a suddenly applied thermal loading, thermal deformation and the role of inertia become more significant.

The determination of the state of stress in the materials of the earth due to the presence of certain sources in the interior of the earth is of great importance in the field of geomechanics, soil mechanics, etc. Here, in the present investigation, we studied the general plane strain problem of an orthotropic micropolar thermoelastic half-space with one relaxation time. Integral transform techniques have been used to solve it. The transformed components of normal stress, tangential stress, tangential couple stress, and temperature distribution in an orthotropic micropolar thermoelastic solid due to mechanical and thermal sources (concentrated/distributed/moving) have been determined.

2 Basic Equations

The basic equations in the dynamic theory of a homogeneous, orthotropic micropolar generalized thermoelastic solid with one relaxation time in the absence of body forces, body couples, and heat sources following Iesan [22] and Dhaliwal and Singh [27] can be expressed as

$$t_{ji,j} = \rho \ddot{u}_i, \tag{1}$$

$$m_{ik,i} + \varepsilon_{ijk} t_{ij} = \rho j \ddot{\phi}_k, \quad i,\ j,\ k = 1,\ 2,\ 3. \tag{2}$$

and the heat conduction equation is given by

$$K_1^* \frac{\partial^2 T}{\partial x_1^2} + K_2^* \frac{\partial^2 T}{\partial x_2^2} = \rho C^* \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) + T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left(\beta_1 \frac{\partial u_1}{\partial x} + \beta_2 \frac{\partial u_2}{\partial y} \right). \tag{3}$$

The constitutive relations are

$$\begin{aligned} t_{11} &= A_{11}\varepsilon_{11} + A_{12}\varepsilon_{22} - \beta_1 T, \quad t_{12} = A_{77}\varepsilon_{12} + A_{78}\varepsilon_{21}, \\ t_{22} &= A_{12}\varepsilon_{11} + A_{22}\varepsilon_{22} - \beta_2 T, \quad t_{21} = A_{78}\varepsilon_{12} + A_{88}\varepsilon_{21}, \\ m_{13} &= B_{66}\phi_{3,1}, \quad m_{23} = B_{44}\phi_{3,2}, \end{aligned} \tag{4}$$

where

$$\varepsilon_{ij} = u_{j,i} + \varepsilon_{ji3}\phi_3. \tag{5}$$

Here the relation between β_i and the coefficients of linear thermal expansion α_i are given by

$$\beta_1 = A_{11}\alpha_1 + A_{12}\alpha_2, \quad \beta_2 = A_{21}\alpha_1 + A_{22}\alpha_2.$$

In the above relations, we have used the following notations:

ρ is the density, j is the microinertia, t_{ij} are the components of the stress sensor, m_{ij} are the components of the couple stress tensor, u_i are the components of the displacement vector, ϕ_k are the components of the microrotation vector, ε_{ij} are the components of the micropolar strain tensor, τ_0 is the relaxation time, ε_{ijk} is the permutation symbol, $A_{11}, A_{12}, A_{77}, A_{78}, A_{88}, B_{44}, B_{66}$ are the characteristic constants of the material, K_1^* and K_2^* are the thermal conductivities, and C^* is the specific heat at constant strain.

3 Formulation of the Problem

We consider a homogeneous, orthotropic micropolar generalized thermoelastic medium in an undisturbed state, initially at a uniform temperature T_o. We introduce the rectangular Cartesian coordinate system (x_1, x_2, x_3) which has its origin on the surface $x_2 = 0$ with the x_2 -axis pointing normally into the medium. Since we are considering the two-dimensional problem parallel to the x_1x_2 -plane, the components of the displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$ can be expressed by

$$\vec{u} = (u_1, u_2, 0), \quad \vec{\phi} = (0, 0, \phi_3) \tag{6}$$

The displacement components u_1, u_2, microrotation component ϕ_3, and temperature distribution T depend upon x_1, x_2, and t and are independent of x_3 coordinate, so that $\frac{\partial}{\partial x_3} \equiv 0$.

To facilitate the solution, the following dimensionless quantities are introduced:

$$(x'_1, x'_2) = \frac{\omega^*(x_1, x_2)}{c_1}, \quad (u'_1, u'_2) = \frac{\rho c_1 \omega^*(u_1, u_2)}{\beta_1 T_0}, \quad t'_{ij} = \frac{t_{ij}}{\beta_1 T_0},$$
$$m'_{23} = \frac{\omega^* m_{23}}{c_1 \beta_1 T_0}, \quad \phi'_3 = \frac{\rho c_1^2 \phi_3}{\beta_1 T_0}, \quad T' = \frac{T}{T_0}, \quad \tau'_1 = \omega^* \tau_1,$$
$$t' = \omega^* t, \quad \tau'_0 = \omega^* \tau_0, \quad P'_1 = \frac{P_1}{\beta_1 T_o}, \quad P'_2 = \frac{P_2}{T_o},$$

where

$$\omega^* = \frac{\rho C^* c_1^2}{K_1^*}, \quad c_1^2 = \frac{A_{11}}{\rho}. \tag{7}$$

With these considerations and using Eqs. 4– 7, Eqs. 1–3 take the form (on suppressing the prime),

$$\left(\frac{\partial^2}{\partial x_2^2} + d_1 d_4 \frac{\partial^2}{\partial x_1^2}\right) u_1 + (d_2 + d_3) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} - (d_3 - 1) \frac{\partial \phi_3}{\partial x_2}$$
$$-d_1 d_4 \frac{\partial T}{\partial x_1} = d_1 d_4 \frac{\partial^2 u_1}{\partial t^2}, \tag{8}$$
$$\frac{(d_2 + d_3)}{d_4} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \left(\frac{\partial^2}{\partial x_2^2} + \frac{d_5}{d_4} \frac{\partial^2}{\partial x_1^2}\right) u_2 - \frac{(d_5 - d_3)}{d_4} \frac{\partial \phi_3}{\partial x_1}$$
$$-\bar{\beta} d_1 \frac{\partial T}{\partial x_2} = d_1 \frac{\partial^2 u_2}{\partial t^2}, \tag{9}$$
$$d_7 (d_3 - 1) \frac{\partial u_1}{\partial x_2} + d_7 (d_5 - d_3) \frac{\partial u_2}{\partial x_1}$$
$$+\left(\frac{\partial^2}{\partial x_2^2} + d_6 \frac{\partial^2}{\partial x_1^2} - d_7 (d_5 - 2d_3 + 1)\right) \phi_3 = d_8 \frac{\partial^2 \phi_3}{\partial t^2}, \tag{10}$$
$$\left(\frac{\partial^2}{\partial x_1^2} + \bar{K} \frac{\partial^2}{\partial x_2^2}\right) T = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T$$
$$+\varepsilon \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) \left(\frac{\partial u_1}{\partial x_1} + \bar{\beta} \frac{\partial u_2}{\partial x_2}\right), \tag{11}$$

where

$$d_1 = \frac{A_{11}}{A_{22}}, \quad d_2 = \frac{A_{12}}{A_{88}}, \quad d_3 = \frac{A_{78}}{A_{88}}, \quad d_4 = \frac{A_{22}}{A_{88}}, \quad d_5 = \frac{A_{77}}{A_{88}}, \quad d_6 = \frac{B_{66}}{B_{44}},$$
$$d_7 = \frac{A_{88} c_1^2}{B_{44} \omega^{*2}}, \quad d_8 = \frac{\rho j c_1^2}{B_{44}}, \quad \bar{K} = \frac{K_2^*}{K_1^*}, \quad \bar{\beta} = \frac{\beta_2}{\beta_1}, \quad \varepsilon = \frac{\beta_1^2 T_0}{\rho K_1^* \omega^*}.$$

Applying Laplace and Fourier transforms defined by

$$\bar{f}(x_1, x_2, p) = \int_0^\infty f(x_1, x_2, t)\mathrm{e}^{-pt}\mathrm{d}t, \tag{12}$$

$$\tilde{f}(\xi, x_2, p) = \int_{-\infty}^\infty \bar{f}(x_1, x_2, t)\mathrm{e}^{i\xi x_1}\mathrm{d}x_1. \tag{13}$$

to Eqs. 8–11, under the assumptions of initial conditions $u_1(x_1, x_2, 0) = \left\{\frac{\partial u_1}{\partial t}\right\}_{t=0} = 0,$

$$u_2(x_1,\ x_2, 0) = \left\{\frac{\partial u_2}{\partial t}\right\}_{t=0} = 0,$$

$$\phi_3(x_1,\ x_2, 0) = \left\{\frac{\partial \phi_3}{\partial t}\right\}_{t=0} = 0, \quad T(x_1,\ x_2, 0) = \left\{\frac{\partial T}{\partial t}\right\}_{t=0} = 0,$$

and assuming $u_1,\ u_2,\ \phi_3,\ T$ and their first-order partial derivatives with respect to x_1 tend to zero as $x_2 \to \pm\infty$, we obtain

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x_2^2} - h\right)\tilde{u}_1 - i\xi(d_2 + d_3)\frac{\mathrm{d}\tilde{u}_2}{\mathrm{d}x_2} - e\frac{\mathrm{d}\tilde{\phi}_3}{\mathrm{d}x_2} + i\xi d_1 d_4 \tilde{T} = 0, \tag{14}$$

$$\frac{-i\xi(d_2 + d_3)}{d_4}\frac{\mathrm{d}\tilde{u}_1}{\mathrm{d}x_2} + \left(\frac{\mathrm{d}^2}{\mathrm{d}x_2^2} - a_{11}\right)\tilde{u}_2 + \frac{i\xi(d_5 - d_3)}{d_4}\tilde{\phi}_3 - \bar{\beta}d_1\frac{\mathrm{d}\tilde{T}}{\mathrm{d}x_2} = 0, \tag{15}$$

$$d_7(d_3 - 1)\frac{\mathrm{d}\tilde{u}_1}{\mathrm{d}x_2} - d_7 i\xi(d_5 - d_3)\tilde{u}_2 + \left(\frac{\mathrm{d}^2}{\mathrm{d}x_2^2} - a\right)\tilde{\phi}_3 = 0, \tag{16}$$

$$-i\xi\varepsilon\, g\, \tilde{u}_1 - \varepsilon\bar{\beta}g\frac{\mathrm{d}\tilde{u}_2}{\mathrm{d}x_2} + \left(\frac{\mathrm{d}^2}{\mathrm{d}x_2^2} - f\right)\tilde{T} = 0. \tag{17}$$

where

$$a = \xi^2 d_6 + d_7(d_5 - 2d_3 + 1) + d_8 p^2, \quad b = \xi^2\frac{(d_2 + d_3)(d_2 + d_4)}{d_4}, \quad e = d_3 - 1,$$

$$f = \frac{\xi^2 + p + \tau_0 p^2}{\bar{K}}, \quad g = (1 + \tau_1 p)d_1\left(\frac{p + n_0\tau_0 p^2}{\bar{K}}\right), \quad h = \left(\xi^2 + p^2\right)d_1 d_4,$$

$$a_{11} = d_1 p^2 + \frac{\xi^2 d_5}{d_4}.$$

The system has a non-trivial solution if and only if the determinant of the factor matrices vanishes. This yields

$$\left(\frac{\mathrm{d}^8}{\mathrm{d}x_2^8} + A\frac{\mathrm{d}^6}{\mathrm{d}x_2^6} + B\frac{\mathrm{d}^4}{\mathrm{d}x_2^4} + C\frac{\mathrm{d}^2}{\mathrm{d}x_2^2} + D\right)\left(\tilde{u}_1, \tilde{u}_2, \tilde{\phi}_3, \tilde{T}\right) = 0, \tag{18}$$

where

$$A = -f - a - a_{11} + \bar{\beta}^2 g - h + b + d_7 e^2,$$
$$B = f\left[a + a_{11} + h - b - d_7 e^2\right] + g\left[-\bar{\beta}^2\left(a + h + d_7 \varepsilon e^2\right) - \varepsilon\xi^2\bar{\beta}(2d_2 + d_3)\right]$$
$$+a(a_{11} + h - b) - a' + p^2 h d_1 - a_{11} d_7 e^2 - \frac{\xi^e d_7(d_3 - d_4)(d_5 - d_3)}{d_4},$$
$$C = f\left\{a_{11} d_7 e^2 + \left[-a_{11} - h + 2b - 2hd_1 p^2 - \xi^2(2d_2 + d_3 + d_4)\right]\right\} + g\{ah\bar{\beta}^2$$
$$+\xi^2 a\varepsilon\bar{\beta}(d_4 - d_3) - 2\bar{\beta}\varepsilon\xi^2 d_7 e(d_5 - d_3) + \varepsilon\xi^2 d_4 a_{11}\} + (a + h)a',$$
$$D = \left(fh - \xi^2 g d_4 \varepsilon\right)\left(a_{11} a + a'\right), \quad a' = \frac{\xi^2 d_7 (d_5 - d_3)^2}{d_4}.$$

The solution of Eq. 18 satisfying the radiation condition that $\tilde{u}_1, \tilde{u}_2, \tilde{\phi}_3, \tilde{T} \rightarrow 0$ as $x_2 \rightarrow \infty$ is

$$\left(\tilde{u}_1, \tilde{u}_2, \tilde{\phi}_3, \tilde{T}\right) = \sum_{i=1}^{4} A_i (1, r_i, s_i, t_i) e^{-q_i x_2}, \tag{19}$$

whereA_i $(i = 1, \ldots, 4)$ are the arbitrary constants to be determined using the boundary conditions and $\pm q_i$ $(i = 1, \ldots, 4)$are the roots of Eq. 18 satisfying

$$\sum_{i=1}^{4} q_i^2 = -A, \quad \sum_{i=1}^{4} q_i^2 q_j^2 = B, \quad \sum_{i=1}^{4} q_i^2 q_j^2 q_k^2 = -C, \quad q_1^2 q_2^2 q_3^2 q_4^2 = D,$$
$$r_i = \frac{a_1 q_i^5 + a_2 q_i^3 + a_3 q_i}{a_4 q_i^4 + a_5 q_i^2 + a_6}, \quad s_i = \frac{-a_1 q_i^3 + r_i\left(a_{10} q_i^2 - a_{11}\right) + a_9 q_i}{a_7 q_i^2 - a_8},$$
$$a_1 = \frac{\bar{\beta}}{i\xi d_4}, \quad t_i = \frac{i\xi s_i q_i e - q_i^2 + h + i\xi r_i q_i (d_2 + d_3)}{i\xi d_1 d_4},$$
$$a_2 = \frac{-\bar{\beta} h + \xi^2(d_2 + d_3) - a\bar{\beta} + d_7 e^2}{i\xi d_4},$$
$$a_3 = \frac{a\left(\bar{\beta} h - \xi^2(d_2 + d_3) - \xi^2 d_7 e(d_5 - d_3)\right)}{i\xi d_4},$$
$$a_4 = 1 - \frac{\bar{\beta}(d_2 + d_3)}{d_4}, \quad a_6 = a a_{11} - a',$$
$$a_5 = -a_{11} - a a_4 - \frac{\bar{\beta} e(d_5 - d_3) d_7}{d_4}, \quad a_7 = \frac{\bar{\beta} e}{i\xi d_4},$$
$$a_8 = \frac{i\xi(d_5 - d_3)}{d_4}, \quad a_9 = \frac{\bar{\beta} h - \xi^2(d_2 + d_3)}{i\xi d_4}, \quad a_{10} = a_4.$$

4 Boundary Conditions

The boundary conditions on the surface $x_2 = 0$ are given as

$$\begin{aligned} &\text{(i)} \quad t_{22} = -P_1\psi(x_1)\eta(t), \\ &\text{(ii)} \quad t_{21} = 0, \\ &\text{(iii)} \quad m_{23} = 0, \\ &\text{(iv)} \quad T = P_2\eta(x_1)\delta(t), \end{aligned} \tag{20}$$

where P_1 is the magnitude of force and P_2 is the constant temperature applied on the boundary; also, $\psi(x_1)$ and $\eta(x_1)$are the known functions and $\delta(t)$ is the Dirac delta function whose Laplace transform with respect to 't' is

$$\bar{\delta}(p) = 1. \tag{21}$$

Using Eq. 7 and then applying Laplace and Fourier transforms from Eqs. 12 and 13 to the system of Eq. 20, and with the help of Eq. 19, we obtained the values of arbitrary constants $A_i (i = 1, \ldots, 4)$. Using these values, and with the help of Eqs. 4 and 5, we get the transformed components of normal stress, tangential stress, tangential couple stress, and temperature distribution as

$$\tilde{t}_{22} = \frac{1}{\Delta}\left(\Delta_1 c_1^* e^{-q_1x_2} + \Delta_2 c_2^* e^{-q_2x_2} + \Delta_3 c_3^* e^{-q_3x_2} + \Delta_4 c_4^* e^{-q_4x_2}\right), \tag{22}$$

$$\tilde{t}_{21} = \frac{1}{\Delta}\left(\Delta_1 a_1^* e^{-q_1x_2} + \Delta_2 a_2^* e^{-q_2x_2} + \Delta_3 a_3^* e^{-q_3x_2} + \Delta_4 a_4^* e^{-q_4x_2}\right), \tag{23}$$

$$\tilde{m}_{23} = \frac{1}{\Delta}\left(\Delta_1 b_1^* e^{-q_1x_2} + \Delta_2 b_2^* e^{-q_2x_2} + \Delta_3 b_3^* e^{-q_3x_2} + \Delta_4 b_4^* e^{-q_4x_2}\right), \tag{24}$$

$$\tilde{T} = \frac{1}{\Delta}\left(\Delta_1 t_1 e^{-q_1x_2} + \Delta_2 t_2 e^{-q_2x_2} + \Delta_3 t_3 e^{-q_3x_2} + \Delta_4 t_4 e^{-q_4x_2}\right), \tag{25}$$

where

$$\begin{aligned}
a_i^* &= \frac{-d_3 i\xi r_i - q_i - e s_i}{d_1 d_4}, \quad b_i^* = -\frac{q_i s_i}{d_1 d_4 d_7}, \\
c_i^* &= \frac{-d_2 i\xi r_i - q_i r_i d_4 - \bar{\beta} t_i d_1 d_4}{d_1 d_4}, \quad i = 1, 2, 3, 4. \\
\Delta &= \left(c_1^* a_2^* - c_2^* a_1^*\right)\left(b_3^* s_4 - s_3 b_4^*\right) + \left(c_3^* a_1^* - c_1^* a_3^*\right)\left(b_2^* s_4 - s_2 b_4^*\right) \\
&\quad + \left(c_1^* a_4^* - c_4^* a_1^*\right)\left(b_2^* s_3 - s_2 b_3^*\right) + \left(c_2^* a_3^* - c_3^* a_2^*\right)\left(b_1^* s_4 - s_1 b_4^*\right) \\
&\quad + \left(c_4^* a_2^* - c_2^* a_4^*\right)\left(b_1^* s_3 - s_1 b_3^*\right) + \left(c_3^* a_4^* - c_3^* a_4^*\right)\left(b_1^* s_2 - s_1 b_2^*\right), \\
\Delta_1 &= -\tilde{P}_1\tilde{\psi}_1(\xi)\left[a_2^*\left(b_3^* s_4 - s_3 b_4^*\right) - a_3^*\left(b_2^* s_4 - s_2 b_4^*\right) + a_4^*\left(b_2^* s_3 - s_2 b_3^*\right)\right] \\
&\quad + \tilde{P}_2\tilde{\eta}(\xi)\left[c_2^*\left(b_3^* s_4 - s_3 b_4^*\right) - c_3^*\left(b_2^* s_4 - s_2 b_4^*\right) + c_4^*\left(b_2^* s_3 - s_2 b_3^*\right)\right], \\
\Delta_2 &= \tilde{P}_1\tilde{\psi}_1(\xi)\left[a_1^*\left(b_3^* s_4 - s_3 b_4^*\right) - a_3^*\left(b_1^* s_4 - s_1 b_4^*\right) + a_4^*\left(b_1^* s_3 - s_1 b_3^*\right)\right] \\
&\quad - \tilde{P}_2\tilde{\eta}(\xi)\left[c_1^*\left(b_3^* s_4 - s_3 b_4^*\right) - c_3^*\left(b_1^* s_4 - s_1 b_4^*\right) + c_4^*\left(b_1^* s_3 - s_1 b_3^*\right)\right],
\end{aligned}$$

$$\begin{aligned}\Delta_3 &= -\tilde{P}_1\tilde{\psi}_1(\xi)\left[a_1^*\left(b_2^*s_4 - s_2b_4^*\right) - a_2^*\left(b_1^*s_4 - s_1b_4^*\right) + a_4^*\left(b_1^*s_2 - s_1b_2^*\right)\right]\\ &\quad+\tilde{P}_2\tilde{\eta}(\xi)\left[c_1^*\left(b_2^*s_4 - s_2b_4^*\right) - c_2^*\left(b_1^*s_4 - s_1b_4^*\right) + c_4^*\left(b_1^*s_2 - s_1b_2^*\right)\right],\\ \Delta_4 &= \tilde{P}_1\tilde{\psi}_1(\xi)\left[a_1^*(b_2^*s_3 - s_2b_3^*) - a_2^*(b_1^*s_3 - s_1b_3^*) + a_3^*(b_1^*s_2 - s_1b_2^*)\right]\\ &\quad+\tilde{P}_2\tilde{\eta}(\xi)\left[-c_1^*(b_2^*s_3 - s_2b_3^*) + c_2^*(b_1^*s_3 - s_1b_3^*) - c_3^*(b_1^*s_2 - s_1b_2^*)\right]. \end{aligned} \tag{26}$$

When $P_2 = 0$, we obtain the solutions for the mechanical force, and when $P_1 = 0$, we obtain the solutions for the thermal source.

5 Applications

5.1 Mechanical Force

5.1.1 Concentrated Normal Force

In order to determine the stresses and temperature distribution due to a concentrated force described as a Dirac delta function, we use $\psi(x_1) = \delta(x_1)$ with

$$\tilde{\psi}(\xi) = 1. \tag{27}$$

5.1.2 Uniformly Distributed Normal Force

The solution due to force distributed over a strip load of dimensionless width $2a$ applied at the plane boundary $x_2 = 0$ is obtained by setting

$$\psi(x_1) = H(x_1 + a) - H(x_1 - a), \tag{28}$$

in Eq. 20. Applying the Fourier transform defined by Eq. 13 to Eq. 28, we obtain

$$\tilde{\psi}(\xi) = \frac{2\,\sin(\xi a)}{\xi}, \qquad \xi \neq 0. \tag{29}$$

5.1.3 Moving Normal Force

To obtain the solution due to an impulsive force, moving along the x_1 axis with a uniform dimensionless speed V at $x_2 = 0$, we use the boundary conditions defined by Eq. 20 by replacing condition (i) with

$$t_{22} = -P_1 H(t)\delta(x_1 - Vt). \tag{30}$$

Using Eqs. 4–6 and then applying Laplace and Fourier transforms defined by Eqs. 12 and 13 to Eq. 30, we obtain

$$\tilde{t}_{22} = \frac{-P_1}{(p - i\xi V)}. \tag{31}$$

The expressions for the stresses and temperature distribution can be obtained in the cases of concentrated and distributed normal forces by substituting the value of $\tilde{\psi}(\xi)$ from Eqs. 27 and 29 and in the case of a moving force by

$$\frac{-P_1}{(p - i\xi V)},$$

in Eq. 26.

5.2 Thermal Source

5.2.1 Concentrated Thermal Source

To determine the normal stress, tangential stress, tangential couple stress, and temperature distribution due to a concentrated source described by the Dirac delta function, $\eta(x_1) = \delta(x_1)$ should be used with

$$\tilde{\eta}(\xi) = 1. \tag{32}$$

5.2.2 Uniformly Distributed Thermal Source

The solution due to a force distributed over a strip load of dimensionless width $2a$ applied on the half-space is obtained by setting

$$\eta(x_1) = H(x_1 + a) - H(x_1 - a), \tag{33}$$

in Eq. 20. Applying the Fourier transform defined by Eq. 13 to Eq. 33, we obtain

$$\tilde{\eta}(\xi) = \frac{2\,\sin(\xi a)}{\xi}, \quad \xi \neq 0. \tag{34}$$

5.2.3 Moving Thermal Source

In this case, boundary condition (iv) in Eq. 20 takes the form

$$T = P_2 H(t)\,\delta(x_1 - Vt). \tag{35}$$

Using Eqs. 4–7 and then applying Laplace and Fourier transforms defined by Eqs. 12 and 13 to Eq. 35, we obtain

$$\tilde{T} = \frac{P_2}{(p - i\xi V)}. \tag{36}$$

The expressions for the stresses and temperature distribution can be obtained for concentrated and distributed thermal sources by substituting the value of $\tilde{\eta}(\xi)$ from Eqs.

32 and 34 and in the case of a moving thermal source by

$$\frac{P_2}{(p - i\xi V)},$$

in Eq. 26.

6 Particular Cases

(i) Neglecting the thermal effect, we obtain the corresponding expressions in the case of the orthotropic micropolar solid and our results agree with those obtained by Kumar et al. [23].

(ii) Neglecting the effect of orthotropy, i.e., by taking

$$\begin{aligned} &A_{11} = A_{22} = \lambda + 2\mu + K, \quad A_{77} = A_{88} = \mu + K, \quad A_{12} = \lambda, \\ &A_{78} = \mu, \quad -K_1 = K_2 = \frac{\chi}{2} = K, \end{aligned}$$

we obtain the corresponding expressions in the case of a micropolar generalized thermoelastic solid and our results agree with those obtained by Kumar et al. [28] in the case of the concentrated mechanical normal force and thermal source.

7 Inversion of the Transforms

To obtain the solution to the problem in the physical domain, we must invert the transforms in Eqs. 22–25. The transformed stresses and temperature distribution are functions of x_2, the parameters of Laplace and Fourier transforms p and ξ, respectively, and hence are of the form $\tilde{f}(\xi,\ x_2,\ p)$.To get the function in the physical domain, we first invert the Fourier transform using

$$\begin{aligned} \bar{f}(x_1, x_2, p) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(\xi, x_2, p)\mathrm{e}^{-i\xi x_1}\mathrm{d}\xi \\ &= \frac{1}{\pi}\int_{0}^{\infty} \left[\bar{f}_\mathrm{e}\cos(\xi, x_2) - i\,\bar{f}_0\sin(\xi, x_2)\right]\mathrm{d}\xi, \end{aligned} \tag{37}$$

where f_e and f_o are, respectively, even and odd parts of the function $\tilde{f}(\xi,\ x_2,\ p)$. Thus, Eq. 37 gives us the Laplace transform $\tilde{f}(\xi,\ x_2,\ p)$ of the function $f(x_1,\ x_2,\ t)$. Following Honig and Hirdes [29], the Laplace transform function $\tilde{f}(\xi,\ x_2,\ p)$can be inverted to $f(x_1,\ x_2,\ t)$. The last step is to calculate the integral in Eq. 37. The method for evaluating this integral is described by Press et al. [30]. It involves the use of Rhomberg's integration with an adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8 Numerical Results and Discussion

Numerical computations are carried out by taking an aluminum epoxy material, subjected to mechanical and thermal disturbances. The physical constants for the orthotropic micropolar thermoelastic solid used by us are:

$$\begin{aligned}
&A_{11} = 13.97 \times 10^9\,\mathrm{N\cdot m^{-2}}, \quad A_{77} = 3.0 \times 10^9\,\mathrm{N\cdot m^{-2}},\\
&A_{88} = 3.2 \times 10^9\,\mathrm{N\cdot m^{-2}}, \quad A_{22} = 13.75 \times 10^9\,\mathrm{N\cdot m^{-2}},\\
&A_{12} = 8.13 \times 10^9\,\mathrm{N\cdot m^{-2}}, \quad A_{78} = 2.2 \times 10^9\,\mathrm{N\cdot m^{-2}},\\
&B_{44} = 0.056 \times 10^5\,\mathrm{N}, \quad B_{66} = 0.056 \times 10^5\,\mathrm{N}.
\end{aligned}$$

Following Gauthier [31], the physical constants for the isotropic micropolar thermoelastic solid are taken as:

$$\begin{aligned}
&\rho = 2.19 \times 10^3\,\mathrm{kg\cdot m^{-3}}, \quad \lambda = 7.59 \times 10^9\,\mathrm{N\cdot m^{-2}}, \quad \mu = 1.89 \times 10^9\,\mathrm{N\cdot m^{-2}},\\
&K = 0.0149 \times 10^9\,\mathrm{N\cdot m^{-2}}, C^* = 0.23\,\mathrm{J\cdot Kg^{-1}\cdot K^{-1}},\\
&\gamma = 0.0268 \times 10^5\,\mathrm{N}, \quad j = 0.196 \times 10^{-4}\,\mathrm{m^2}.
\end{aligned}$$

The comparison of the normal stress, tangential couple stress, and temperature distribution for an orthotropic micropolar thermoelastic solid (OMTS) and isotropic micropolar thermoelastic solid (IMTS) is shown in Figs. 1, 2, 3, 4, 5, 6, and 7. The computations were carried out at $x_2 = 0.1$ within the range $0 \le x_1 \le 10$. The curves represented by solid lines with or without a center symbol correspond to the case of OMTS whereas the curves represented by dotted lines with or without a center symbol correspond to the case of IMTS. All the results are shown for one value of a dimensionless width $a = 0.04$ and two values of dimensionless time $t = 0.1$ and 0.2. In Figs. 1, 2, and 3, solid and dotted lines without a center symbol represent the variations due to a concentrated source (C) whereas solid and dotted lines with center symbol $(-0-0-)$ represent the variations due to a uniformly distributed source (UD). In Figs. 4, 5, and 6 , solid and dotted lines without a center symbol represent the variations at $t = 0.1$, whereas solid and dotted lines with center symbol $(-0-0-)$ represent the variations at $t = 0.2$.

Figure 1 shows the variation of normal stress t_{22} with respect to distance x. For OMTS, on the application of both concentrated and uniformly distributed forces, the value of the normal stress t_{22} starts with a sharp increase within the range $0 \le x_1 \le 2$ and then oscillates with a decreasing amplitude, whereas for IMTS its value initially increases and then oscillates about the zero value.

The variation of $m_{23}$with respect to distance x is shown in Fig. 2. In the case of OMTS, on the application of both concentrated and uniformly distributed forces, the value of m_{23} oscillates with a decreasing amplitude; however, for IMTS, its value oscillates with increasing magnitude.

It is observed from Fig. 3 that the variation of the temperature distribution T, for OMTS, decreases and then oscillates with very small oscillations about the zero value, while for IMTS, the curve shows the oscillatory behavior. The values of t_{22}, m_{23}, and T for both orthotropic and isotropic solids on the application of a concentrated force

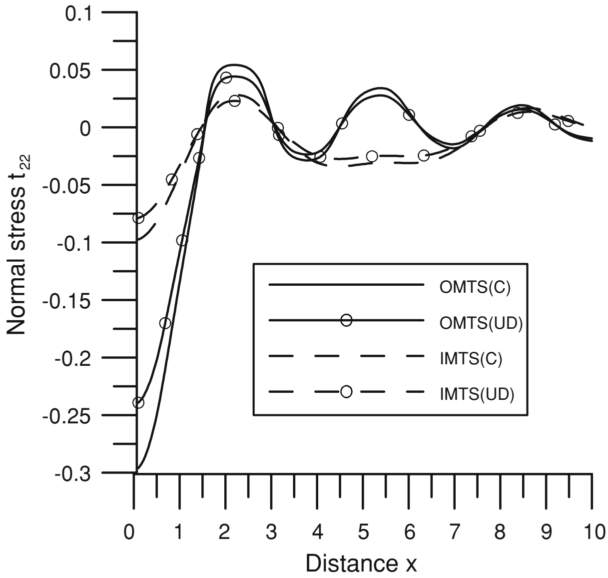

Fig. 1 Variation of the normal stress t_{22} with respect to distance x due to mechanical force

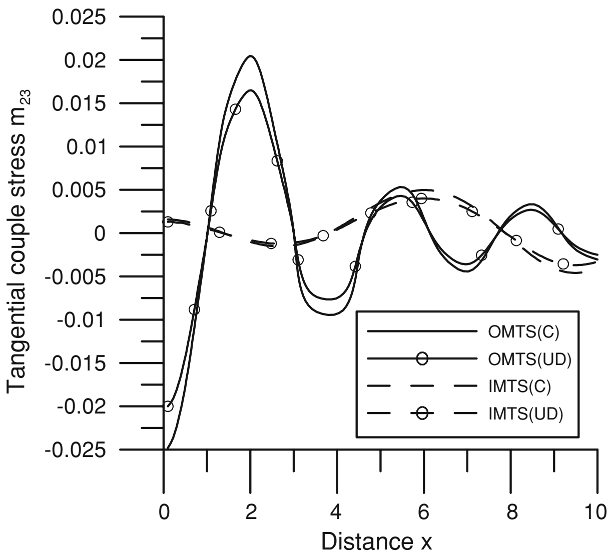

Fig. 2 Variation of the tangential couple stress m_{23} with respect to distance x due to mechanical force

are shown in the figure by dividing their original values by 10. Figures 4, 5, and 6 show the variations in the values of the normal stress, tangential couple stress, and temperature distribution due to a moving thermal source.

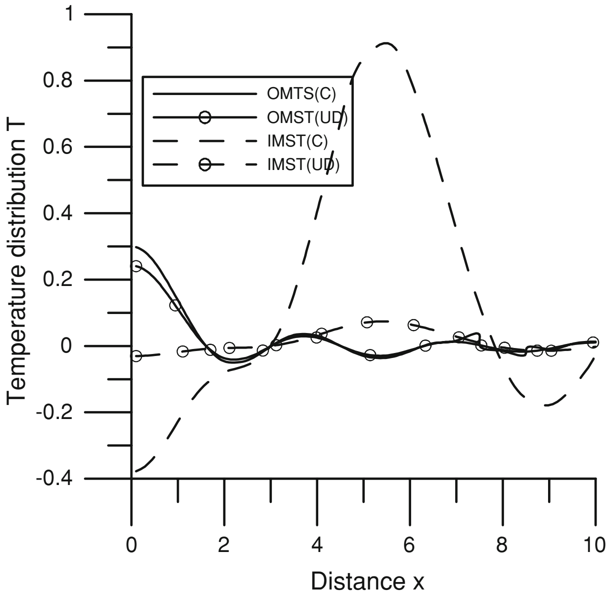

Fig. 3 Variation of the temperature distribution T with respect to distance x due to mechanical force

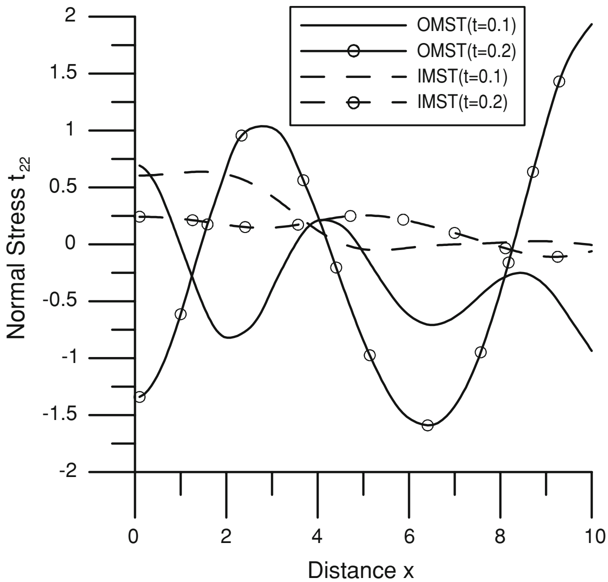

Fig. 4 Variation of the normal stress t_{22} with respect to distance x due to moving thermal source

From Fig. 4 it is observed that the values of t_{22} at $t = 0.1$ initially decrease and then oscillate with a decreasing amplitude, while at $t = 0.2$, its value oscillates with

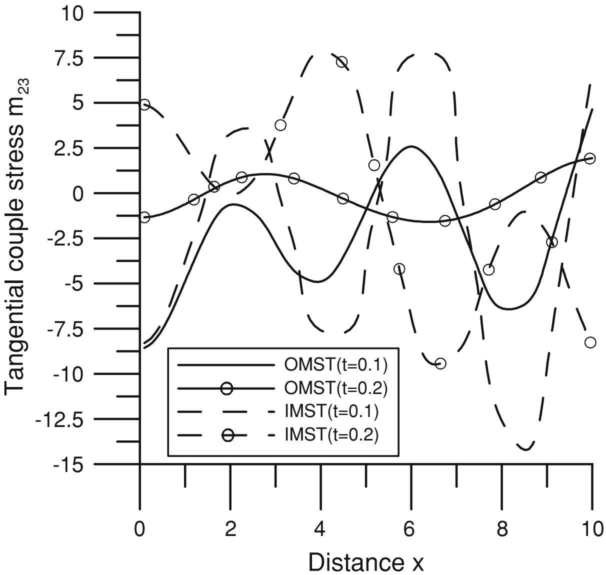

Fig. 5 Variation of the tangential couple stress $m_{23}$with respect to distance x due to moving thermal source

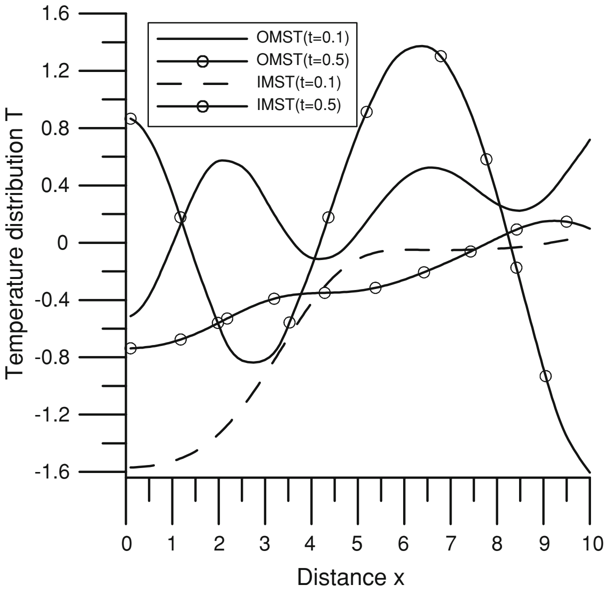

Fig. 6 Variation of the temperature distribution T with respect to distance x due to moving thermal source

a very large amplitude, in the case of OMTS. However, for IMTS its value decreases and approaches zero for both the values $t = 0.1$ and $t = 0.2$.

Figure 5 depicts the variation of the tangential couple stress m_{23} with respect to distance x due to a moving thermal source. In the case of OMTS and IMTS, the value

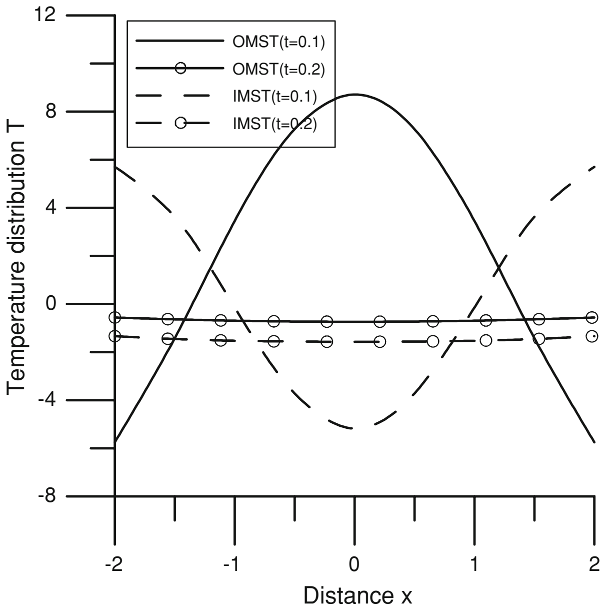

Fig. 7 Variation of the temperature distribution T with respect to distance x

of tangential couple stress increase or decrease, respectively, with distance x for both $t = 0.1$ and $t = 0.2$. Its value decreases with an increase in anisotropy.

It is evident from Fig. 6 that the variation of the temperature distribution is exactly opposite to the variation of t_{22} for OMTS while in the case of IMTS, its value increases with an increase in distance x. In the figure showing the temperature distribution T, the graph shows the negative values of T, which in fact refer to the small temperature change. It is decreasing in nature, then reaches zero, and then again starts decreasing in agreement with the theoretical results. Also in Fig. 7 the variations of T with distance x at two different values of $t$are shown within the range $-2 \leq x_1 \leq 2$. The variation of T for OMTS and IMTS at $t = 0.1$ is exactly opposite in nature, while at $t = 0.2$, its behavior is similar for OMTS and IMTS. Similar observations have also been made by Ezzat et al. [32] and Chandrasekhariah and Srinath [33].

9 Conclusion

A significant anisotropy effect has been observed on the normal stresst_{22} tangential couple stress m_{23} and temperature distribution T. From the above discussion, it is evident that variations of stresses and temperature distribution on the application of different forces follow a similar trend and behavior except for a slight variation in their amplitudes. Initially, the magnitude of stress on application of a uniformly distributed force is large in comparison to that obtained when a concentrated force is applied, but afterward it becomes larger for a concentrated force in both cases, orthotropic and isotropic. However, for a temperature distribution, reverse behavior is observed. It is also observed that, when a moving thermal source is applied, the values of stresses and T increase for an orthotropic micropolar thermoelastic solid with an increase in time, while the behavior is opposite for an isotropic micropolar thermoelastic solid. With an increase in anisotropy, the values of the normal stress and tangential couple stress decrease while the reverse behavior is observed for a temperature distribution.

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